2. S. I. Plaksin, "On stationary solutions of the motion equations of a fluid with gas bubbles," Prikl. Mekh. Tekh. Fiz., No. 1 (1983).
3. V. Yu. Lyapidevskii and S. I. Plaksin, "Shock structure in a gas—fluid meidum with a nonlinear equation of state," Dinamika Sploshnoi Sredy, No. 62, Inst. Gidrodin., Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1983).
4. P. Bkhatnagar, Nonlinear Waves in One-Dimensional Disperse Systems [Russian translation], Mir, Moscow (1983).
5. G. B. Whitham, Linear and Nonlinear Waves, Wiley, New York (1974).

STABILITY OF A DUSTY NONISOTHERMAL GAS JET

E. P. Kurochkina and M. P. Strongin UDC 532.517.6.013.4

Interest in modeling the behavior of gas-dispersed flows with large parameter gradients has increased greatly in recent years for several reasons. On the one hand, there are an increasing number of practical uses for such flows. Examples of this are found in the chemical industry and in the area of environmental protection (propagation of aerosols). On the other hand, the interest also stems from improved possibilities for calculating such flows. In this regard, investigators are especially attracted to the problem of the stability of gas-dispersed flows. The solution of this problem would in several cases make it possible to obtain estimates of the critical parameters corresponding to the transition from laminar to turbulent flow. Calculations of stability performed in [1-3] for dusty isothermal gas flows showed that a flow may be appreciably stabilized by particles (the critical Reynolds numbers may increase by several orders of magnitude under certain conditions). No calculations have been made of the stability of thermally stratified gas flows with a disperse phase, although such calculations would most likely have practical value. Here, we examine the stability of a dusty plane jet with a temperature differing considerably from the medium in which the jet is flowing.

The flow of a submerged viscous nonisothermal gas-dispersed jet is described by the system of Navier–Stokes equations with allowance for the gas-particle interaction, which is modeled by a term of the Stokes force type. As was noted in [1-3], an important parameter is $\beta = \tau/\tau_0$, where $\tau = L/(U_m \alpha C)$ (L and U_m are the characteristic scales of length and velocity of the jet, while α and C are the wave number and the phase velocity of the perturbations). The quantity $\tau_0 = \rho_0 d^2(18\mu_g)$ is the time of Stokes relaxation relative to the particle velocity (ρ_0 is the density of the particle material, d is the particle diameter, and μ_g is the viscosity of the gas). The case $\beta \ll 1$ is usually realized in actual dusty flows. The following evaluations can serve as an illustration. For particles of the diameter 10^{-4} m and density $\rho \approx 10^4$ kg/m³ with a hot-air viscosity $\mu_g \approx 2\cdot10^{-5}$ kg/(m·sec), the relaxation time is $\tau_0 \approx 5/18$ sec. At the same time, for typical jet scales $L \approx 10^{-2}$ m, $U_m \approx 2\cdot10^2$ m/sec, and $\alpha C \approx 10^{-2}$ (from the results of our study), $\tau \approx 5\cdot10^{-3}$ sec and $\beta \approx 18\cdot10^{-3}$. Thus, the charcteristic fluctuation velocities of the particles are considerably less than the fluctuation velocity of the gas. As a result, in the analysis of stability presented here, the disturbance of the particles can be ignored. Since the parameter $\beta_1 = 18\rho_g L^2/(\rho_0 \mathrm{Re}\, d^2)$ depends on the Reynolds number $\mathrm{Re} = LU_m/\nu_g$ ($\nu_g^{-1} = \rho_g/\mu_g$, ρ_g is the density of the gas), it is convenient to use it as an independent varible (in [2-4], β was assigned; this led to obvious problems in calculating neutral curves with $\beta_1 \ll 1$). As the characteristic parameter in the present study, we take $A = 18\delta(L/d)^2$ (δ is the volumetric concentration of particles). For the above parameters, $18(L/d) \approx 1.8\cdot10^5$ and with a change in δ from 10^{-5} to 10^{-2}, A may incrase from 1.8 to $1.8\cdot10^3$.

Proceeding on the basis of the Navier–Stokes equations for a nonisothermal flow and using Stokes' law to describe the effect of the particles on the gas flow, we can obtain the following system of equations:

$$\rho\frac{dU}{dt} = -\frac{\partial P}{\partial x} + \frac{2}{\mathrm{Re}_-}\frac{\partial}{\partial x}\left(\mu\frac{\partial U}{\partial x}\right) + \frac{1}{\mathrm{Re}_-}\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)\right] - \frac{2}{3\mathrm{Re}_-}\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)\right] + \frac{An\mu}{\mathrm{Re}_-}(U - U_0),$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 97-101, September-October, 1988. Original article submitted June 16, 1987.

 0021-8944/88/2905-0690$12.50

$$\rho\frac{dV}{\partial t}=-\frac{\partial P}{\partial y}+\frac{2}{\mathrm{Re}_-}\frac{\partial}{\partial y}\left(\mu\frac{\partial V}{\partial y}\right)+\frac{1}{\mathrm{Re}_-}\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\right]-\frac{2}{3\,\mathrm{Re}_-}\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}\right)\right]+\frac{An\mu}{\mathrm{Re}_-}(V-V_0),$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho U)}{\partial x}+\frac{\partial(\rho V)}{\partial y}=0.$$

Here and below, U, V, and x, y are the longitudinal and transverse velocities and the coordinates; P is pressure; n is the distribution of particle concentration; $\rho=\rho_g/\rho_{g-}$; $\mu=\mu_g/\mu_{g-}$; $\mathrm{Re}_-=LU_m/\nu_{g-}$; the plus subscript corresponds to values of the parameters on the jet axis, while the minus subscript corresponds to values at infinity with respect to the y coordinate. A subscript of zero denotes particle velocity. We will study the stationary solutions of this system in a plane-parallel approximation. The fields of velocity, temperature, and concentration are assumed to be given and are independent of y. For the velocity field, we take step functions, corresponding to the initial section of the jet, and we use relations of the similarity type:

$$U_1=1,\ |y|<1;\ U_1=0,\ |y|\geqslant 1; \tag{1}$$

$$U_1=\{1+\mathrm{th}[(1-|y|)/\theta]\}/2,\ \theta\ll 1,\ -\infty<y<+\infty; \tag{2}$$

$$U_1=1-\mathrm{th}^2 y,\ -\infty<y<+\infty, \tag{3}$$

while $T(y)=sU_1(y)+1$, $n(y)=-U_1(y)$. The parameter s varies from 0 to −0.9 (cold flow) and from 0 to 10 (hot). The flow is isothermal at s = 0. It is assumed that density is inversely proportional to temperature, while the relation $\mu=\sqrt{T}$ is used for viscosity. We will ignore fluctuations of temperature and particle concentration.

Using the small perturbation method [5], we obtain the system of equations

$$i\alpha\,\mathrm{Re}_-\rho(U_1-C)u+\rho\,\mathrm{Re}_-U_1'v=-i\alpha\,\mathrm{Re}_-p-4\alpha^2\mu u/3-2i\alpha\mu v'/3+i\alpha(\mu v)'+(\mu u')'+An\mu u; \tag{4}$$

$$i\alpha\mathrm{Re}_-\rho(U_1-C)v=-\mathrm{Re}_-p'-\alpha^2\mu v-2i\alpha(\mu u)'/3+i\alpha\mu u'+4(\mu v')'/3+An\mu v; \tag{5}$$

$$i\alpha\rho u+(\rho v)'=0, \tag{6}$$

where u(y), v(y), p(y) are the amplitudes of the velocity and pressure perturbations. We will examine solutions which are symmetrical on the axis and which decay at infinity. The mathematical formulation of these conditions is presented below. The critical Reynolds number Re_* is found as the minimum $\mathrm{Re}_+=\mathrm{Re}_-\rho_+/\mu_+$ for neutral perturbations. Due to the symmetry of the problem, we can restrict ourselves to positive y. The stability of a one-phase isothermal flow was studied in [6] for (1), while the stability of a two-phase flow was examined for (2), (3), in [1, 7, 8].

For the profile (1), system (4)-(6) has piecewise-constant coefficients. The region over the y values can be subdivided into two subregions ($0\leq y<1$ and $1\leq y<+\infty$), and we can write an analytical solution in each of these subregions. The fact that the functions U_1, ρ, and μ have discontinuities makes it difficult to analyze system (4)-(6). Thus, in accordance with the laws of mass and momentum conservation, we first introduce the new variables $\varphi=\rho v$ (used in (6)), $q=4\mu v'/3-2i\alpha\mu u/3-\mathrm{Re}_-p$ (from Eq. (5)), and $w=\mathrm{Re}_-U_1\varphi+i\alpha\mu v/3-\mu u'$. We determine w by means of (4) and the relation $\mu v'=2(\mu v)'/3-i\alpha\mu u/3$, which follows from the temperature dependences of density and viscosity adopted above. The new variables are also continuous for the discontinuous profiles of U_1 and T.

In the new variables, system (4)-(6) has the following form (the variable u remains the same):

$$\begin{aligned}u'&=[\mathrm{Re}_-U_1/\mu+i\alpha/(3\rho)]\varphi-w/\mu,\\ q'&=[4\alpha^2\mu/(3\rho)-i\alpha\mathrm{Re}_-C-An\mu/\rho]\varphi+i\alpha w,\\ \varphi'&=-i\alpha\rho u,\\ w'&=[i\alpha\mathrm{Re}_-\rho(C-2U_1)-8\alpha^2\mu/3+An\mu]u+i\alpha q.\end{aligned} \tag{7}$$

We seek a solution which decays at infinity and which is symmetrical at zero with respect to the functions φ and w:

$$\begin{aligned}u_+&=c_1\,\mathrm{sh}(\alpha y)+c_2\,\mathrm{sh}(\varkappa y),\\ q_+&=-i(\alpha+\varkappa^2/\alpha)\mu_+c_1\,\mathrm{sh}(\alpha y)-2i\alpha\mu_+c_2\,\mathrm{sh}(\varkappa y),\\ \varphi_+&=-i\rho_+c_1\,\mathrm{ch}(\alpha y)-i\alpha\rho_+c_2\,\mathrm{ch}(\varkappa y)/\varkappa,\end{aligned}$$

$$w_+ = -(i\mathrm{Re}_-\rho_+ + 2\alpha\mu_+/3)c_1 \operatorname{ch}(\alpha y) + (\alpha^2\mu_+/3 - \varkappa^2\mu_+ - i\alpha\,\mathrm{Re}_-\,\rho_+)c_2 \operatorname{ch}(\varkappa y)/\varkappa$$

(where $0 \leqslant y < 1$, $\varkappa^2 = \alpha^2 + i\alpha\mathrm{Re}_-(1-C)\rho_+/\mu_+ - A)$, $u_- = c_3\exp(-\alpha y) + c_4\exp(-\gamma y)$, $q_- = -(\mathrm{Re}_-C + 2i\alpha)c_3\exp(-\alpha y) - 2i\alpha c_4\exp(-\gamma y)$, $\varphi_- = ic_3\exp(-\alpha y) + i\alpha c_4\exp(-\gamma y)/\gamma$, $w_- = 2\alpha c_3\exp(-\alpha y)/3 + [\gamma - \alpha^2/(3\gamma)]c_4\exp(-\gamma y)$ $(1 \leqslant y < +\infty$, $\gamma^2 = \alpha^2 - i\alpha\mathrm{Re}_-C)$. The conditions of continuity of u, q, and w at the point y = 1 lead to the system

$$Fc_m = 0, \tag{8}$$

$$F = \begin{pmatrix} \operatorname{sh}\alpha & \operatorname{sh}\varkappa & -\exp(-\alpha) & -\exp-(\gamma) \\ F_{21} & F_{22} & F_{23} & 2i\alpha\exp(-\gamma) \\ -i\rho_+\operatorname{ch}\alpha & F_{32} & -i\exp(-\alpha) & -i\alpha\exp(-\gamma)/\gamma \\ F_{41} & F_{42} & -2\alpha\exp(-\alpha)/3 & F_{44} \end{pmatrix},$$

$$F_{21} = -i(\alpha + \varkappa^2/\alpha)\mu_+\operatorname{sh}\alpha,\ F_{22} = -2i\,\alpha\mu_+\operatorname{sh}\varkappa,$$

$$F_{23} = (\mathrm{Re}_-C + 2i\alpha)\exp(-\alpha),\ F_{32} = -i\alpha\rho_+\operatorname{ch}\varkappa/\varkappa,$$

$$F_{41} = -(2\alpha\mu_+/3 + i\mathrm{Re}_-\rho_+)\operatorname{ch}\alpha,\ F_{42} = (\alpha^2\mu_+/3 - \varkappa^2\mu_+ - i\alpha\mathrm{Re}_-\rho_+)\operatorname{ch}\varkappa/\varkappa,$$

$$F_{44} = [\alpha^2/(3\gamma) - \gamma]\exp(-\gamma),\ c_m = (c_1, c_2, c_3, c_4)^{\perp}.$$

For (8) to have a nontrivial solution, it is necessary to satisfy the condition

$$\det\|F\| = 0. \tag{9}$$

Using the solution (9) and the secant method, we can construct a neutral curve relative to Re_+ and find Re_*.

System (7) can be solved only numerically for profiles (2) and (3). The use of system (4)-(6) leads to serious complications in the numerical realization due to the presence of coefficients with large gradients (similar to the derivative of a rapidly changing function). To perform numerical calculations in the present study, we will use the Crank–Nicholson method and the technique of counter trial runs. Since $U_1(y)$, $T(y)$, and $n(y)$ approach zero as $y \to \infty$, the decay conditions can be reduced to the following: $u = [(\gamma^2 - \alpha^2/3 - 2\alpha\gamma/3)\varphi + i(\gamma - \alpha)w]/(\alpha\mathrm{Re}_-C)$, $q = \{[(\alpha^2/3 - \gamma^2)(2i\alpha + \mathrm{Re}_-C) + 4i\alpha^2\gamma/3]\varphi - (\gamma - \alpha)^2 w\}/(\alpha\mathrm{Re}_-C)$. These conditions can be formulated at y = 2 for (2) and at y = 6 for (3).

Figure 1 (curves 1 and 2) shows the results of calculations for one-phase thermally stratified jets (A = 0). The values of Re_* of the hot flow decrease (curve 1 corresponds to (1), (2), while curve 2 corresponds to (3)) and the jet is destabilized. With eight fold overheating, the process is slowed for (1) and (2), and Re_* depends less on the amount of overheating. It turns out that hot jets with similarity profile (3) are less stable and that destabilization is slowed with much greater overheating. In the cold jet, we find an exponential (with respect to T_+) increase in Re_* $(\mathrm{Re}_* \sim T_+^{-1}$ for (3) and $\mathrm{Re}_* \sim T_+^{-0.83}$ for (1), (2)). Meanwhile, the flow with profile (3) is more stable than the flows with profiles (1), (2) for $\log T_+ < 0$.

The behavior of the critical wave numbers for the nonisothermal flows is interesting (Fig. 2). These numbers have a maximum, the maximum being found in the overheated region (curve 1) for profiles of the stepped type (1), (2) and in the cold region for (3) (curve 2). The critical phase velocities C_* also have a maximum (Fig. 3), but their behavior is more complex (curve 1 corresponds to (1), (2), while curve 2 corresponds to (3)). To study the effect of variability of the thermophysical parameters of the flows on Re_*, we performed calculations with the constant ρ and μ. It was found that a change in density has the greatest effect on Re_*. This is quite natural, since it is more dependent on temperature.

The results of studies for two-phase jets with profile (1) are shown in Fig. 1 (lines 3 and 4) and Fig. 4. Since the hot jets are less stable than the isothermal or cold jets, we will examine hot dusty flows. At $T_+ = 11$, we obtained a dependence of Re_* on the parameter A (curve 2 in Fig. 4) that was proportional to the volumetric concentration of particles. For A > 30, Re_* is linearly dependent on A ($\mathrm{Re}_* = 0.5A + 32$), but stabilization is much weaker than in the isothermal case (curve 1). Figure 1 (lines 3 and 4) shows the dependence of Re_* on T_+ for different A (lines 3 and 4 correspond to A = 23 and 50). The value of Re_* changes little, even with a sevenfold increase in temperature. The calculations performed for profiles (1) and (2) nearly coincide. Since we used different methods to calculate their stability, this served as a cross check.

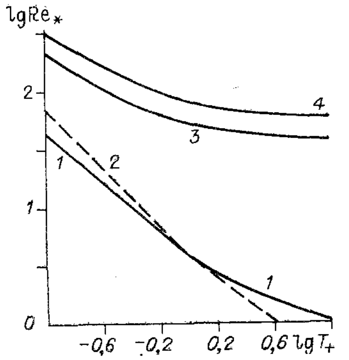

Fig. 1

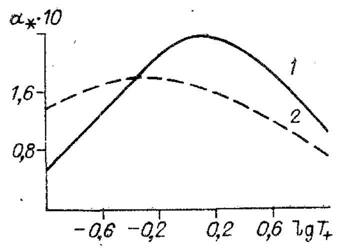

Fig. 2

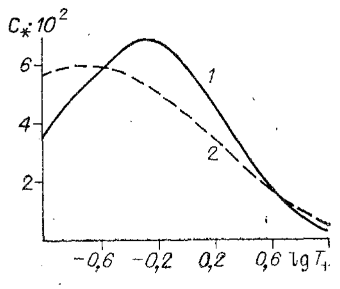

Fig. 3

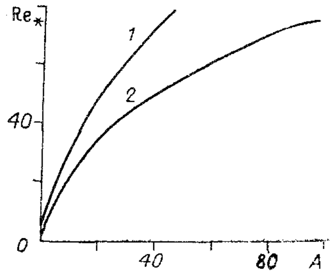

Fig. 4

Thus, stabilization of the flows is seen with an increase in particle concentration. The heating of the jet in a certain manner (curves 3 and 4 in Fig. 1 and curve 2 in Fig. 4) extinguishes the stabilization effect without disturbing the linear dependence of Re_* on A, i.e., as in the isothermal case, it was shown that the jet can be stabilized appreciably by the particles (Re_* may increase by several orders of magnitude). Also, we observed that the high-temperature jet with the stepped profile was much more stable than the jet with a similarity profile. In the isothermal case, the values of Re_* coincide.

LITERATURE CITED

1. E. P. Kurochkina and M. P. Strongin, "Nonlinear stability of two-phase jets," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1985).
2. P. G. Saffman, "On the stability of laminar flow of a dusty gas," J. Fluid Mech., 13, No. 1 (1962).
3. D. H. Michael, "The stability of plane Poiseuille flow of a dusty gas," J. Fluid Mech., 18, No.1 (1964).
4. Ch. B. Narmuratov and A. S. Solov'ev, "Effect of suspended particles on the stability of a plane Poiseuille flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1986).
5. M. A. Gol'dshtik and V. N. Shtern, Hydrodynamic Stability and Turbulence, Nauka, Novosibirsk (1977).
6. E. P. Kurochkina, "Regime shift in a plane jet with a stepped velocity profile," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1984).
7. E. P. Kurochkina, "Stability and branching of plane two-phase jet flows," in: Stability and Turbulence [in Russian], ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1985).
8. E. P. Kurochkina, "Calculation of the nonlinear stability of two-phase flows," in: Turbulent Two-Phase Flows and Experimental Methods [in Russian], Pt. 2, Inst. Termofiz. Elektrofiz. Akad. Nauk EstSSR, Tallin (1985).